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Radiation and self-polarization of neutral fermions in quasi-classical description

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Abstract
A Lorentz invariant formalism for the quasi-classical description of electromagnetic radiation from a neutral spin $\frac{1}{2}$ particle with an anomalous magnetic moment moving in an external electromagnetic field is developed. In the high symmetry fields, for which analytical solutions to the Bargmann–Michel–Telegdi equation are known, the so-called self-polarization axes, i.e. directions of preferred polarization of particles in the radiation process, are found. Expressions for the radiative transition probability and spectral-angular distribution of the radiation emitted by a polarized particle are obtained in the fields under consideration.

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1. Introduction

Electromagnetic radiation from an uncharged spin $\frac{1}{2}$ particle with an anomalous magnetic moment moving in certain types of classical electromagnetic field was studied previously within the Furry picture of quantum electrodynamics (QED) [1–8]. Meanwhile it was observed that under certain conditions the radiation process may be described in purely classical terms using the Bargmann–Michel–Telegdi (BMT) spin evolution equation [9, 10][1]. In such a pseudoclassical treatment the radiation power is given by the well-known formula for a magnetic moment radiation [12]:

$$\frac{\mathrm{d}I}{\mathrm{d}O} = -\frac{1}{4\pi(lu)^5u^0}\{\ddot{\mu}^\nu\ddot{\mu}_\nu(ul)^2 + (l^\nu\ddot{\mu}_\nu)^2\}, \tag{1}$$

where μ^ν is the 4-vector of the magnetic moment and u^ν is the 4-velocity of the particle. Here $l^\nu = \{1, \mathbf{l}\}$, where $\mathbf{l}$ is the unit vector in the direction of an emitted wave; a dot denotes the differentiation with respect to the proper time τ. We use the units $\hbar = c = 1$.

[1] For more details, see [11] and references therein.

The radiation power calculated within the framework of QED is found to correspond to the result obtained from equation (1) under the conditions of quasi-classical character of the motion, namely, that the binding energy due to the magnetic moment in the rest frame should be much smaller than the particle mass, and the external field should vary slowly at the distances of the order of the Compton length, which amounts to the conditions:

$$\mu_0 H_0 \ll mc^2, \qquad \hbar \dot{H}_0/mc^2 H_0 \ll 1, \tag{2}$$

where H_0 is the magnetic field strength in the rest frame and μ_0 is the value of particle magnetic moment (here we use Gaussian units).

In our papers [13, 14] the radiation of unpolarized neutral particles was investigated in the quasi-classical approach in the case of an arbitrary field. To study radiation from an unpolarized particle one must impose an additional requirement that consists in averaging over the initial spin states and summing over the final polarizations. The averaging of the quantum transition amplitudes should correspond to the averaging over the initial orientations of the magnetic dipole moment within the quasi-classical consideration. We proposed [13, 14] to replace the magnetic moment by

$$\mu^\nu = \mu_0 S^\nu, \tag{3}$$

where S^ν is the mean value of the spin vector, its evolution being described by the BMT equation [15]:

$$\dot{S}^\nu = 2\mu_0\{F^{\nu\alpha}S_\alpha - u^\nu(u_\alpha F^{\alpha\beta}S_\beta)\}. \tag{4}$$

The validity of this equation is ensured by conditions (2) [16, 17].

Our main goal was to show that when the averaging over polarization states at $\tau = \tau_0$ is performed, the resulting expression for the radiation power depends only on the external field intensity and thus is valid in the case of an arbitrary external field subjected to conditions (2). It is important that the neutral particle moves with a constant velocity in the external field. Of course, the true quantum description of radiation demands the accounting of quantum recoil in the photon emission process. But when conditions (2) are satisfied, the energies of emitted photons are small; therefore, we can neglect the change of the particle velocity.

Unfortunately, while studying the radiation of polarized particles, even with the assumptions similar to those discussed above, the approach based on formula (1) is valid only for the transitions without spin flip. So here we will use another method [18] based on the introduction of quasi-classical spin wavefunctions in QED formulae.

Such wavefunctions can be constructed as follows [19]. Suppose the Lorentz equation is solved, i.e. the dependence of coordinates of the particle on proper time is found. Then the BMT equation transforms to ordinary differential equation, resolvent of which determines a one-parametric subgroup of the Lorentz group. The quasi-classical spin wavefunction signifies a spin-tensor, whose evolution is determined by the same one-parametric subgroup.

It is easy to verify that for a neutral particle with spin $\frac{1}{2}$, represented by a Dirac bispinor, the equation for the wavefunction $\Psi(\tau)$ under consideration is [19]

$$\dot{\Psi} = \mathrm{i}\mu_0\gamma^5 H^{\mu\nu}u_\nu\gamma_\mu\hat{u}\Psi, \tag{5}$$

where $H^{\mu\nu} = -\frac{1}{2}\,\mathrm{e}^{\mu\nu\rho\lambda}F_{\rho\lambda}$ is the dual electromagnetic tensor. Obviously, the density matrix of the partially polarized fermion takes the form

$$\rho(\tau,\tau') = \tfrac{1}{2}U(\tau,\tau_0)(\hat{p}(\tau_0)+m)(1-\gamma^5\hat{S}(\tau_0))U^{-1}(\tau',\tau_0), \tag{6}$$

where $U(\tau,\tau_0)$ is the resolvent of equation (5). For a pure state the density matrix reduces to the direct product of bispinors, normalized by the condition $\bar{\Psi}(\tau)\Psi(\tau) = 2m$.

2. General relations

Now let us investigate the radiation of polarized particles on the basis of the above considerations. The formula of quantum electrodynamics which describes the transition probability of a neutral fermion under spontaneous radiation in an external field is[2]:

$$P = -\int \mathrm{d}^4x\,\mathrm{d}^4y \int \frac{\mathrm{d}^4p\,\mathrm{d}^4q\,\mathrm{d}^4k}{(2\pi)^6}\delta(k^2)\delta(p^2-m^2)\delta(q^2-m^2) \\ \times \varrho^{\mu\nu}_{ph}(x,y;k)\,\mathrm{Sp}\{\Gamma_\mu(x)\varrho_i(x,y;p)\Gamma_\nu(y)\varrho_f(y,x;q)\}. \tag{7}$$

Here $\varrho_i(x, y; p)$, $\varrho_f(y, x; q)$ are density matrices of the initial (i) and the final (f) states of the fermion, $\varrho^{\mu\nu}_{ph}(x, y; k)$ is the density matrix of the radiated photon, $\Gamma^\mu = -\sqrt{4\pi}\mu_0\sigma^{\mu\nu}k_\nu$ is the vertex function.

In order to pass to the quasi-classical approximation, it is necessary to substitute precise density matrices for those constructed in [19][3] (see (6)) and to neglect the recoil in the photon emission process. The latter operation implies inserting the following expression:

$$(2\pi)^3\frac{p^0q^0}{m^2}\iint \mathrm{d}\tau\,\mathrm{d}\tau'\delta^4(x^\alpha-u^\alpha\tau)\delta^4(y^\beta-u^\beta\tau')\delta(\mathbf{p}-m\mathbf{u})\delta(\mathbf{q}-m\mathbf{u}), \tag{8}$$

in the integrand of (7), which reduces the integration to the particle trajectory. After summation over photon polarizations and integration with respect to fermion momenta and coordinates, we obtain the quasi-classical expressions for the transition probability under investigation:

$$P = \frac{\mu_0^2}{(2\pi)^2}\int \mathrm{d}O\int_0^\infty k^3\,\mathrm{d}k\iint \mathrm{d}\tau\,\mathrm{d}\tau'\,\mathrm{e}^{\mathrm{i}k(lu)(\tau-\tau')}T(\tau,\tau';u), \tag{9}$$

and for the radiation energy

$$\mathcal{E} = \frac{\mu_0^2}{(2\pi)^2}\int \mathrm{d}O\int_0^\infty k^4\,\mathrm{d}k\iint \mathrm{d}\tau\,\mathrm{d}\tau'\,\mathrm{e}^{\mathrm{i}k(lu)(\tau-\tau')}T(\tau,\tau';u). \tag{10}$$

Here the following notation is introduced:

$$T(\tau,\tau';u) = (V_iV_f - A_iA_f), \tag{11}$$

where V_{if}, A_{if} are determined by formulae:

$$\begin{aligned} V_i &= \tfrac14\mathrm{Sp}\{\hat{l}U(\tau)(1+\hat{u})(1-\gamma^5\hat{S}_{0i})U^{-1}(\tau')\},\\ V_f &= \tfrac14\mathrm{Sp}\{\hat{l}U(\tau')(1+\hat{u})(1-\gamma^5\hat{S}_{0f})U^{-1}(\tau)\},\\ A_i &= \tfrac14\mathrm{Sp}\{\gamma^5\hat{l}U(\tau)(1+\hat{u})(1-\gamma^5\hat{S}_{0i})U^{-1}(\tau')\},\\ A_f &= \tfrac14\mathrm{Sp}\{\gamma^5\hat{l}U(\tau')(1+\hat{u})(1-\gamma^5\hat{S}_{0f})U^{-1}(\tau)\}. \end{aligned} \tag{12}$$

In formulae (9) and (10) the operator $U(\tau) \equiv U(\tau, \tau_0)$ is the resolvent of equation (5). Naturally, the spectral-angular distributions for the transition probability and radiation energy can be obtained only if the solution of equation (5) is known. So first we concentrate on investigating the general properties of these physical values.

Let us introduce the integration variable $k' = k(lu)$, which denotes the photon energy in the rest frame of the radiating particle. Using formulae for the Fourier transforms of generalized functions (for example, see [20]), we integrate (9) and (10) with respect to variable k'. As a

[2] In the expression for the radiation energy $\mathcal{E}$ the additional multiple k—the energy of radiated photon—appears in the integrand.

[3] Note, that for arbitrary plane-wave fields such a substitution is identical.

result, we obtain

$$P = \frac{\mu_0^2}{(2\pi)^2} \int \frac{\mathrm{d}O}{(lu)^4} \iint \mathrm{d}\tau\, \mathrm{d}\tau' \frac{1}{2(\tau - \tau' + \mathrm{i}0)} \left(\partial_\tau \partial_{\tau'}^2 - \partial_\tau^2 \partial_{\tau'}\right) T(\tau, \tau'; u), \tag{13}$$

$$\mathcal{E} = \frac{\mu_0^2}{(2\pi)^2} \int \frac{\mathrm{d}O}{(lu)^5} \iint \mathrm{d}\tau\, \mathrm{d}\tau' \frac{\mathrm{i}}{(\tau - \tau' + \mathrm{i}0)} \partial_\tau^2 \partial_{\tau'}^2 T(\tau, \tau'; u). \tag{14}$$

Expressions (13) and (14) could be integrated with respect to the angular variables. As $U(\tau)$ is the resolvent of equation (5), one has

$$\int \frac{\mathrm{d}O}{(lu)^4} T(\tau, \tau'; u) \to \frac{16\pi}{3} \tilde{V}_i \tilde{V}_f, \qquad \int \frac{\mathrm{d}O}{(lu)^5} T(\tau, \tau'; u) \to \frac{16\pi}{3} u^0 \tilde{V}_i \tilde{V}_f, \tag{15}$$

where

$$\tilde{V}_i = \tfrac{1}{4}\mathrm{Sp}\{U(\tau)(1 - \gamma^5 \hat{S}_{0i}) U^{-1}(\tau')\}, \qquad \tilde{V}_f = \tfrac{1}{4}\mathrm{Sp}\{U(\tau')(1 - \gamma^5 \hat{S}_{0f}) U^{-1}(\tau)\}. \tag{16}$$

Thus

$$P = \frac{4\mu_0^2}{3\pi} \iint \mathrm{d}\tau\, \mathrm{d}\tau' \frac{1}{2(\tau - \tau' + \mathrm{i}0)} \left(\partial_\tau \partial_{\tau'}^2 - \partial_\tau^2 \partial_{\tau'}\right) \tilde{V}_i \tilde{V}_f, \tag{17}$$

$$\mathcal{E} = \frac{4\mu_0^2 u^0}{3\pi} \iint \mathrm{d}\tau\, \mathrm{d}\tau' \frac{\mathrm{i}}{(\tau - \tau' + \mathrm{i}0)} \partial_\tau^2 \partial_{\tau'}^2 \tilde{V}_i \tilde{V}_f. \tag{18}$$

Obviously, function $T(\tau, \tau'; u)$ does not vary if $\tau \leftrightarrow \tau'$ and $S_{0i}^\mu \leftrightarrow S_{0f}^\mu$. It gives the possibility of obtaining the following expressions for the angular distribution:

$$\left.\frac{\mathrm{d}I}{\mathrm{d}O}\right|_{S_{0i} \to S_{0f}} + \left.\frac{\mathrm{d}I}{\mathrm{d}O}\right|_{S_{0f} \to S_{0i}} = \frac{\mu_0^2}{2\pi u^0 (lu)^5} \partial_\tau^2 \partial_{\tau'}^2 T(\tau, \tau'; u)\Big|_{\tau = \tau'}, \tag{19}$$

and for the total radiation power

$$I_{S_{0i} \to S_{0f}} + I_{S_{0f} \to S_{0i}} = \frac{8\mu_0^2}{3} \partial_\tau^2 \partial_{\tau'}^2 \tilde{V}_i \tilde{V}_f \Big|_{\tau = \tau'}. \tag{20}$$

If we denote the solutions of the BMT equation with initial conditions $S_i^\mu(\tau_0) = S_{0i}^\mu$ and $S_f^\mu(\tau_0) = S_{0f}^\mu$ as S_i^μ and S_f^μ we obtain

$$\begin{aligned}
\left.\frac{\mathrm{d}I}{\mathrm{d}O}\right|_{S_{0i} \to S_{0f}} + \left.\frac{\mathrm{d}I}{\mathrm{d}O}\right|_{S_{0f} \to S_{0i}} = {} & \frac{\mu_0^2}{2\pi u^0 (lu)^5} \big\{2(lu)^2 [4H^2((H^2) + (HS_i)(HS_f)) \\
& - (\dot{H}^2 + (\dot{H}S_i)(\dot{H}S_f))] - 4(lu)\big[(HS_i) e_{\mu\nu\rho\lambda} l^\mu \dot{H}^\nu H^\rho S_f^\lambda \\
& + (HS_f) e_{\mu\nu\rho\lambda} l^\mu \dot{H}^\nu H^\rho S_i^\lambda\big] - 2[(4H^2 (Hl)^2 - (\dot{H}l)^2) \\
& + 4H^2 (Hl) e_{\mu\nu\rho\lambda} l^\mu \dot{H}^\nu u^\rho H^\lambda] - 2[(lS_i)(lS_f)(4(H^2)^2 + \dot{H}^2) \\
& + 8(Hl)^2 (HS_i)(HS_f)] + 8H^2 (Hl)[(lS_i)(HS_f) + (lS_f)(HS_i)] \\
& + 2(\dot{H}l)[(lS_i)(\dot{H}S_f) + (lS_f)(\dot{H}S_i)] - 4H^2\big[(lS_i) e_{\mu\nu\rho\lambda} l^\mu \dot{H}^\nu u^\rho S_f^\lambda \\
& + (lS_f) e_{\mu\nu\rho\lambda} l^\mu \dot{H}^\nu u^\rho S_i^\lambda\big] - 8H^2 e_{\mu\nu\rho\lambda} l^\mu H^\nu u^\rho S_i^\lambda e_{\mu\nu\rho\lambda} l^\mu H^\nu u^\rho S_f^\lambda \\
& - 2 e_{\mu\nu\rho\lambda} l^\mu \dot{H}^\nu u^\rho S_i^\lambda e_{\mu\nu\rho\lambda} l^\mu \dot{H}^\nu u^\rho S_f^\lambda - 4((lS_i)(H\dot{H}) - (Hl)(\dot{H}S_i) \\
& - (\dot{H}l)(HS_i)) e_{\mu\nu\rho\lambda} l^\mu H^\nu u^\rho S_f^\lambda - 4((lS_f)(H\dot{H}) - (Hl)(\dot{H}S_f) \\
& - (\dot{H}l)(HS_f)) e_{\mu\nu\rho\lambda} l^\mu H^\nu u^\rho S_i^\lambda\big\},
\end{aligned} \tag{21}$$

$$I_{S_{0i}\to S_{0f}} + I_{S_{0f}\to S_{0i}} = \tfrac{16}{3}\mu_0^2\{4H^2((H^2) + (HS_i)(HS_f)) - (\dot{H}^2 + (\dot{H}S_i)(\dot{H}S_f)) - 2[(HS_i)e_{\mu\nu\rho\lambda}u^\mu \dot{H}^\nu H^\rho S_f^\lambda + (HS_f)e_{\mu\nu\rho\lambda}u^\mu \dot{H}^\nu H^\rho S_i^\lambda]\}, \tag{22}$$

where H^μ denotes $\mu_0 H^{\mu\nu}u_\nu$. If we average over the initial spin states and summarize over the final ones in expressions (21) and (22), for which purpose we must set $S_i^\mu = S_f^\mu = 0$, we obtain the angular distribution and total radiation power of unpolarized fermion [13, 14]. If we set $S_i^\mu = S_f^\mu = S^\mu$ in formulae (21) and (22) and divide the obtained expression by 2, we obtain the angular distribution and total radiation power without spin flip. One can see that the radiation powers without spin flip are equal for the states of the particle with opposite polarizations.

Using the above technique, we obtain for the transition probability at the time moment t:

$$\left.\frac{\mathrm{d}W}{\mathrm{d}O}\right|_{S_{0i}\to S_{0f}} - \left.\frac{\mathrm{d}W}{\mathrm{d}O}\right|_{S_{0f}\to S_{0i}} = \frac{\mu_0^2}{4\pi u^0(lu)^4}\{4((HS_i) - (HS_f))(H^2(lu)^2 - (Hl)^2) - (lu)e_{\mu\nu\rho\lambda}l^\mu \dot{H}^\nu H^\rho (S_i^\lambda - S_f^\lambda) + ((lS_i) - (lS_f))e_{\mu\nu\rho\lambda}l^\mu \dot{H}^\nu H^\rho u^\lambda - (Hl)e_{\mu\nu\rho\lambda}l^\mu \dot{H}^\nu u^\rho (S_i^\lambda - S_f^\lambda) + (\dot{H}l)e_{\mu\nu\rho\lambda}l^\mu H^\nu u^\rho (S_i^\lambda - S_f^\lambda)\}, \tag{23}$$

$$W_{S_{0i}\to S_{0f}} - W_{S_{0f}\to S_{0i}} = \frac{4\mu_0^2}{3u^0}\{4H^2((HS_i) - (HS_f)) - e_{\mu\nu\rho\lambda}u^\mu \dot{H}^\nu H^\rho (S_i^\lambda - S_f^\lambda)\}. \tag{24}$$

Let us define the state of total self-polarization $S_{0\mathrm{tp}}^\mu$ by either of the two equivalent conditions:

$$I_{S_{0\mathrm{tp}}\to -S_{0\mathrm{tp}}} = 0, \qquad W_{S_{0\mathrm{tp}}\to -S_{0\mathrm{tp}}} = 0. \tag{25}$$

In the general case, in this state $I_{S_{0\mathrm{tp}}\to S_{0\mathrm{tp}}} \neq 0$. If the state of the total self-polarization $S_{0\mathrm{tp}}^\mu$ is known, then the radiation power of the particle in the $-S_{0\mathrm{tp}}^\mu$ state is

$$I_{-S_{0\mathrm{tp}}} = \tfrac{8}{3}\mu_0^2\{4H^2(3H^2 - (HS_{\mathrm{tp}})^2) - (3\dot{H}^2 - (\dot{H}S_{\mathrm{tp}})^2) + 4(HS_{\mathrm{tp}})e_{\mu\nu\rho\lambda}u^\mu \dot{H}^\nu H^\rho S_{\mathrm{tp}}^\lambda\}. \tag{26}$$

Underline that the obtained expressions do not allow us to decide, which state of the particle is the state with maximum possible radiative self-polarization, also what is the self-polarization degree, and whether the state of total self-polarization exists. This is due to the fact that in the general case the quantity $W_{S_0\to -S_0} + W_{-S_0\to S_0}$ necessary for obtaining the balance equation (see, for example, [21]) could not be expressed in terms of solutions to the BMT equation.

However, we can show that there always exists either the total self-polarization state, or the state with maximum possible self-polarization for a neutral fermion moving in an external field, provided that this field belongs to a class of fields admitted, on one hand, by the requirement that equation (5) and consequently the BMT equation should allow precise analytical solutions, and, on the other hand, specified by conditions of solvability that do not depend on the magnitude of the anomalous moment μ_0. The latter can become the total self-polarization state under continuous variation of the field characteristics.

3. Radiation and self-polarization in the fields of special type

Let us prove the above statement. The conditions for the existence of analytical solutions of the BMT equation and their form are given in [22][4]. Using the results of [22], in order to

[4] The conditions for the existence of analytical solutions of the BMT equation are reduced to condition (34) (see also [23]).

obtain the solutions of (5), we introduce the following basis vectors:

$$
\begin{aligned}
&n_0^\mu = u^\mu, \qquad n_1^\mu = H^\mu/\sqrt{-H^2}, \\
&n_2^\mu = (H^\mu(H\dot{H}) - \dot{H}^\mu H^2)/\sqrt{-H^2 N}, \\
&n_3^\mu = -\mathrm{e}^{\mu\nu\rho\lambda} u_\nu H_\rho \dot{H}_\lambda/\sqrt{N},
\end{aligned} \tag{27}
$$

where

$$
N = H^2 \dot{H}^2 - (H\dot{H})^2. \tag{28}
$$

This basis is orthogonal and its elements satisfy the system of equations, which is a four-dimensional generalization of the Frenet equations:

$$
\dot{n}_1^\mu = \kappa n_2^\mu, \qquad \dot{n}_2^\mu = \varkappa n_3^\mu - \kappa n_1^\mu, \qquad \dot{n}_3^\mu = -\varkappa n_2^\mu. \tag{29}
$$

Here parameters κ and $\varkappa$ are analogous of curvature and torsion:

$$
\kappa = \frac{N^{1/2}}{(-H^2)}, \qquad \varkappa = \frac{\sqrt{-H^2}}{N} \mathrm{e}^{\alpha\beta\gamma\delta} \ddot{H}_\alpha \dot{H}_\beta H_\gamma u_\delta. \tag{30}
$$

In the chosen basis the spin vector is of the form

$$
S^\mu = \sum_{i=1}^{3} S_i n_i^\mu,
$$

where $S_i = -(Sn_i)$ are components of the three-dimensional spin vector $\mathbf{S}$.

The $\mathbf{S}$ components satisfy the set of equations

$$
\dot{S}_1 = \kappa S_2, \qquad \dot{S}_2 = (\varkappa - 2\sqrt{-H^2})S_3 - \kappa S_1, \qquad \dot{S}_3 = -(\varkappa - 2\sqrt{-H^2})S_2. \tag{31}
$$

Let us define $N_i = \gamma^5 \hat{n}_i \hat{u}$. Obviously $N_i N_j = -\mathrm{i} e_{ijk} N_k$, $\bar{N}_i = N_i$. Using standard transition from spinor to bispinor representation [24], we obtain from formulae [22]

$$
U = VR_0, \qquad V = \cos\frac{\Omega_0}{2} + \mathrm{i}(\mathbf{Nt}_0)\sin\frac{\Omega_0}{2}, \qquad R_0 = \cos\frac{\Omega}{2} - \mathrm{i}(\mathbf{Nt})\sin\frac{\Omega}{2}. \tag{32}
$$

Here

$$
\begin{aligned}
&\mathbf{t}_0 = \{\eta_\varkappa, 0, \eta_\kappa\}\left(\eta_\varkappa^2 + \eta_\kappa^2\right)^{-1/2} \\
&\Omega_0 = \left(\eta_\varkappa^2 + \eta_\kappa^2\right)^{1/2} \int_{\tau_0}^{\tau} \sqrt{-H^2}\,\mathrm{d}\tau \\
&\mathbf{t} = \{\eta_\varkappa - 2, 0, \eta_\kappa\}\left((\eta_\varkappa - 2)^2 + \eta_\kappa^2\right)^{-1/2} \\
&\Omega = \left((\eta_\varkappa - 2)^2 + \eta_\kappa^2\right)^{1/2} \int_{\tau_0}^{\tau} \sqrt{-H^2}\,\mathrm{d}\tau,
\end{aligned} \tag{33}
$$

where

$$
\eta_\varkappa = \frac{\varkappa}{\sqrt{-H^2}} = \text{const}, \qquad \eta_\kappa = \frac{\kappa}{\sqrt{-H^2}} = \text{const}. \tag{34}
$$

The meaning of the introduced vectors $\mathbf{t}_0$ and $\mathbf{t}$ is quite evident. The vector $\mathbf{t}_0$ represents the axis about which the trihedron determining the field orientation in the rest frame of the particle rotates (i.e. the Darboux vector). Vector **t** defines the axis about which the spin vector precesses in the particle rest frame.

Note that, if conditions (34) are satisfied, the vector **t** is the only constant solution of system (31). The corresponding solution of the BMT equation is

$$S^\mu = [H^\mu(\varkappa - 2\sqrt{-H^2})/\sqrt{-H^2} - \mathrm{e}^{\mu\nu\rho\lambda} u_\nu H_\rho \dot{H}_\lambda \kappa/\sqrt{N}][(\varkappa - 2\sqrt{-H^2})^2 + \kappa^2]^{-1/2}. \quad (35)$$

The characteristic of this solution is that the spin vector (35) retains the orientation with respect to the external field. As we shall see later, it is precisely this vector that fixes the direction of the self-polarization axis.

Inserting (30) into (16), we obtain

$$\begin{aligned}\tilde{V}_i = &\left\{\cos\frac{\Omega-\Omega'}{2} + \mathrm{i}(\mathbf{tS}_{0i})\sin\frac{\Omega-\Omega'}{2}\right\}\cos\frac{\Omega_0-\Omega_0'}{2} + \left\{(\mathbf{tt}_0)\sin\frac{\Omega-\Omega'}{2}\right.\\ &-\mathrm{i}\left[(\mathbf{tt}_0)(\mathbf{tS}_{0i})\cos\frac{\Omega-\Omega'}{2} + ((\mathbf{S}_{0i}\mathbf{t}) - (\mathbf{tt}_0)(\mathbf{tS}_{0i}))\cos\frac{\Omega+\Omega'}{2}\right.\\ &\left.\left.+(\mathbf{S}_{0i}[\mathbf{t}\times\mathbf{t}_0])\sin\frac{\Omega+\Omega'}{2}\right]\right\}\sin\frac{\Omega_0-\Omega_0'}{2}.\end{aligned} \quad (36)$$

The expression for $\tilde{V}_f$ is obtained from (36) after the substitution $\mathbf{S}_{0i} \to -\mathbf{S}_{0f}$. If we set $\mathbf{S}_{0i} = \zeta_i\mathbf{t}$ and $\mathbf{S}_{0f} = \zeta_f\mathbf{t}$, we obtain

$$\begin{aligned}\tilde{V}_i\tilde{V}_f = &\tfrac{1}{4}\,\mathrm{e}^{\mathrm{i}(\Omega-\Omega')(\zeta_i-\zeta_f)/2}\{[\mathrm{e}^{\mathrm{i}(\Omega_0-\Omega_0')(\zeta_i+\zeta_f)/2} + \mathrm{e}^{-\mathrm{i}(\Omega_0-\Omega_0')(\zeta_i+\zeta_f)/2}](1-(\mathbf{tt}_0)^2)\\ &+\mathrm{e}^{\mathrm{i}(\Omega_0-\Omega_0')(\zeta_i-\zeta_f)/2}(1-(\mathbf{tt}_0))^2 + \mathrm{e}^{-\mathrm{i}(\Omega_0-\Omega_0')(\zeta_i-\zeta_f)/2}(1+(\mathbf{tt}_0))^2\}.\end{aligned} \quad (37)$$

Inserting this expression into (17) and (18), we obtain the transition probability with and without the spin flip, and consequently the total radiation energy. If $\sqrt{-H^2} = \text{const}$, we can obtain final expressions for the above values by locking the integration contour in lower or upper half-plane of the complex variable $\tau - \tau'$. Hence, if $\dot{\Omega} > \dot{\Omega}_0$, we obtain the following expression for the transition probability with spin flip:

$$W_{\zeta_i\to-\zeta_i} = \frac{2\mu_0^2}{3u^0}\{2(\dot{\Omega})^3(1-(\mathbf{tt}_0)^2) + (\dot{\Omega}-\dot{\Omega}_0)^3(1+(\mathbf{tt}_0))^2 + (\dot{\Omega}+\dot{\Omega}_0)^3(1-(\mathbf{tt}_0))^2\}\Theta(-\zeta_i). \quad (38)$$

If $\dot{\Omega} < \dot{\Omega}_0$, then

$$\begin{aligned}W_{\zeta_i\to-\zeta_i} = &\frac{2\mu_0^2}{3u^0}\{[(\dot{\Omega}_0-\dot{\Omega})^3(1+(\mathbf{tt}_0))^2]\Theta(\zeta_i) + [2(\dot{\Omega})^3(1-(\mathbf{tt}_0)^2)\\ &+(\dot{\Omega}+\dot{\Omega}_0)^3(1-(\mathbf{tt}_0))^2]\Theta(-\zeta_i)\}.\end{aligned} \quad (39)$$

Here $\Theta(x)$ is the Heaviside theta function.

Thus if $\dot{\Omega} > \dot{\Omega}_0$ the state with spin vector (35) will be the state with the total self-polarization, otherwise it will have the partial self-polarization. Using (36), it is easy to verify that the maximum possible degree of self-polarization is obtained in the state (35):

$$\frac{W_{\zeta_p\to-\zeta_p} - W_{-\zeta_p\to\zeta_p}}{W_{\zeta_p\to-\zeta_p} + W_{-\zeta_p\to\zeta_p}} = \frac{2(\dot{\Omega})^3(1-(\mathbf{tt}_0)^2) + (\dot{\Omega}-\dot{\Omega}_0)^3(1+(\mathbf{tt}_0))^2 + (\dot{\Omega}+\dot{\Omega}_0)^3(1-(\mathbf{tt}_0))^2}{2(\dot{\Omega})^3(1-(\mathbf{tt}_0)^2) + |\dot{\Omega}-\dot{\Omega}_0|^3(1+(\mathbf{tt}_0))^2 + (\dot{\Omega}+\dot{\Omega}_0)^3(1-(\mathbf{tt}_0))^2}. \quad (40)$$

The transition probability without spin flip is determined as

$$W_{\zeta_i\to\zeta_i} = \frac{2\mu_0^2}{3u^0}\{(\dot{\Omega}_0)^3(1-(\mathbf{tt}_0)^2)\}. \quad (41)$$

The total radiation power is

$$I_{\zeta_i\to-\zeta_i} = \frac{2\mu_0^2}{3}\{2(\dot\Omega)^4(1-(\mathbf{tt}_0)^2) + (\dot\Omega-\dot\Omega_0)^4(1+(\mathbf{tt}_0))^2 + (\dot\Omega+\dot\Omega_0)^4(1-(\mathbf{tt}_0))^2\}\Theta(-\zeta_i) \tag{42}$$

if $\dot\Omega > \dot\Omega_0$ and

$$\begin{aligned} I_{\zeta_i\to-\zeta_i} = \frac{2\mu_0^2}{3}\{&[(\dot\Omega_0-\dot\Omega)^4(1+(\mathbf{tt}_0))^2]\Theta(\zeta_i) \\ &+[2(\dot\Omega)^4(1-(\mathbf{tt}_0)^2) + (\dot\Omega+\dot\Omega_0)^4(1-(\mathbf{tt}_0))^2]\Theta(-\zeta_i)\} \end{aligned} \tag{43}$$

if $\dot\Omega < \dot\Omega_0$;

$$I_{\zeta_i\to\zeta_i} = \frac{2\mu_0^2}{3}\{(\dot\Omega_0)^4(1-(\mathbf{tt}_0)^2)\}. \tag{44}$$

These formulae show that in the fields under investigation, in the rest frame, the particle can radiate photons of only four energies: those corresponding to the characteristic frequency of the external field variation, to the frequency of spin precession and to two combination frequencies, the radiation with the external field frequency being possible only without spin flip.

The obtained formulae may be somewhat simplified using explicit expressions for $\dot\Omega$, $\dot\Omega_0$, $\mathbf{t}_0$ and $\mathbf{t}$. We perform this for special fields.

4. Examples

First of all, we consider the case of constant homogeneous magnetic field. This problem is, in a sense, a test for calculation techniques. It was first discussed in [1] within the framework of quantum theory, and then was repeatedly investigated using various quasi-classical methods [25] (see, also, [26, 27]). Since in the field under consideration $\kappa = \varkappa = 0$, the integrals over τ, τ' in formulae (9), (10) can be calculated precisely for arbitrary S_{0i}, S_{0f}. We obtain

$$\begin{aligned} \left.\frac{\mathrm{d}W}{\mathrm{d}O\,\mathrm{d}k}\right|_{S_{0i}\to S_{0f}} &= \frac{\mu_0^2k^3}{8\pi u^0}\delta(k(lu)-2\sqrt{-H^2}) \\ &\quad\times\left((lu)^2-\frac{(Hl)^2}{H^2}\right)\left(1-\frac{(HS_{0i})}{\sqrt{-H^2}}\right)\left(1+\frac{(HS_{0f})}{\sqrt{-H^2}}\right), \end{aligned} \tag{45}$$

$$\begin{aligned} \left.\frac{\mathrm{d}I}{\mathrm{d}O\,\mathrm{d}k}\right|_{S_{0i}\to S_{0f}} &= \frac{\mu_0^2k^4}{8\pi u^0}\delta(k(lu)-2\sqrt{-H^2}) \\ &\quad\times\left((lu)^2-\frac{(Hl)^2}{H^2}\right)\left(1-\frac{(HS_{0i})}{\sqrt{-H^2}}\right)\left(1+\frac{(HS_{0f})}{\sqrt{-H^2}}\right). \end{aligned} \tag{46}$$

Obviously, in this case, the self-polarization is total, and self-polarization axis is $S^\mu_{\mathrm{tp}} = -H^\mu/\sqrt{-H^2}$, i.e., depending on the sign of an anomalous magnetic moment, in the rest frame the particle spin is oriented either along or opposite to the direction of the magnetic field. Because of the relation $\dot\Omega_0 = 0$, the radiation frequency in the rest frame is equal to the frequency of spin precession. It is highly important that, under transition from any spin state, the spectral-angular distribution of the radiation is the same irrespective of whether the spin flip takes place. In this case the classical formula for magnetic dipole radiation is valid only for the transitions without spin flip. However, due to the above feature, the radiation power calculated using this formula differs from the correct value only in numerical coefficient.

After the integration with respect to the angles and photon energies, we obtain

$$W_{S_{0i}\to S_{0f}} = \frac{16\mu_0^2}{3u^0}(-H^2)^{3/2}\left(1-\frac{(HS_{0i})}{\sqrt{-H^2}}\right)\left(1+\frac{(HS_{0f})}{\sqrt{-H^2}}\right), \tag{47}$$

$$I_{S_{0i}\to S_{0f}} = \frac{32\mu_0^2}{3}(H^2)^2\left(1-\frac{(HS_{0i})}{\sqrt{-H^2}}\right)\left(1+\frac{(HS_{0f})}{\sqrt{-H^2}}\right). \tag{48}$$

If we introduce $S_{0i}^\mu = \zeta_i S_{\mathrm{tp}}^\mu$, $S_{0f}^\mu = \zeta_f S_{\mathrm{tp}}^\mu$ and consider the transitions only between those states, we obtain the expressions analogous to those derived in [1].

Let us now discuss the radiation in the field of a circularly polarized monochromatic plane wave with the frequency ω and the amplitude E. In this case

$$\begin{aligned} &H^\mu = -\mu_0 E\big[\big((nu)a_1^\mu - (a_1u)n^\mu\big)\cos\omega(nx) + g\big(a_2^\mu(nu) - (a_2u)n^\mu\big)\sin\omega(nx)\big],\\ &-H^2 = \mu_0^2(nu)^2E^2, \qquad \kappa = \omega(nu), \qquad \varkappa = 0. \end{aligned} \tag{49}$$

Here

$$\begin{aligned} n^\mu &= (1,\mathbf{n}), \qquad & n_+^\mu &= \tfrac{1}{2}(1,-\mathbf{n}),\\ a_1^\mu &= (0,\mathbf{a}_1), \qquad & a_2^\mu &= (0,\mathbf{a}_2), \end{aligned} \tag{50}$$

where $\omega\mathbf{n}$ is the wave vector and $\mathbf{a}_i$ are the unit vectors of polarization.

Consequently,

$$\dot\Omega_0 = \omega(nu), \qquad \dot\Omega = \omega(nu)(1+d^2)^{1/2}, \qquad (\mathbf{tt}_0) = (1+d^2)^{-1/2}, \tag{51}$$

where $d = 2\mu_0E/\omega$. Since the condition $\dot\Omega_0 < \dot\Omega$ is satisfied for any wave parameters, the transition probability is determined by (38) and (41), and the radiation power by (42) and (44).

It was indicated above that in the particle rest frame the photons with only four frequencies are radiated. Partial transition probabilities and radiation powers are expressed as

$$\begin{aligned} W_{\zeta_i\to-\zeta_i}(\dot\Omega) &= \frac{4\mu_0^2\omega^3(nu)^3d^2}{3u^0}(1+d^2)^{1/2}\Theta(-\zeta_i),\\ W_{\zeta_i\to-\zeta_i}(\dot\Omega\pm\dot\Omega_0) &= \frac{2\mu_0^2\omega^3(nu)^3d^4}{3u^0(1+d^2)}((1+d^2)^{1/2}\pm 1)\Theta(-\zeta_i),\\ W_{\zeta_i\to\zeta_i}(\dot\Omega_0) &= \frac{2\mu_0^2\omega^3(nu)^3d^2}{3u^0(1+d^2)}, \end{aligned} \tag{52}$$

$$\begin{aligned} I_{\zeta_i\to-\zeta_i}(\dot\Omega) &= \frac{4\mu_0^2\omega^4(nu)^4\,\mathrm{d}^2}{3}(1+d^2)\Theta(-\zeta_i),\\ I_{\zeta_i\to-\zeta_i}(\dot\Omega\pm\dot\Omega_0) &= \frac{2\mu_0^2\omega^4(nu)^4d^4}{3(1+d^2)}((1+d^2)^{1/2}\pm 1)^2\Theta(-\zeta_i),\\ I_{\zeta_i\to\zeta_i}(\dot\Omega_0) &= \frac{2\mu_0^2\omega^4(nu)^4d^2}{3(1+d^2)}. \end{aligned} \tag{53}$$

The fact that the particle can radiate not only with the frequency of the external wave, but also with other three frequencies, which are not multiples of the first one, was mentioned in [4]. We must emphasize that in our case the self-polarization axis does not determine a constant space direction, but is rigidly tied to the external field. Namely, in the rest frame of the particle its spin vector precesses with the wave frequency around the wave vector, the spin vector being in the same plane with the wave vector and the vector of magnetic field strength, the angle between the spin vector and the wave vector being equal to the arc tangent of parameter d.

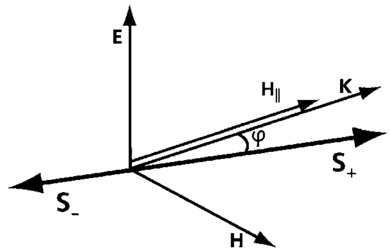

Figure 1. Orientation of the self-polarization axis for the Redmond field.

The interesting case is the Redmond field, which is the superposition of the above discussed circularly polarized wave and a constant homogeneous magnetic field directed along its wave vector. Since in the Redmond field conditions (34) are satisfied only when the particle moves along the constant magnetic field $H_\|$, we study this case and set $(a_i u) = 0$. Then

$$H^\mu = -\mu_0 E(nu)\big[a_1^\mu \cos\omega(nx) + g a_2^\mu \sin\omega(nx)\big] - \mu_0 H_\| \big[n^\mu (n_+ u) - n_+^\mu (nu)\big],$$
$$-H^2 = \mu_0^2\big((nu)^2 E^2 + H_\|^2\big), \tag{54}$$

$$\kappa = \frac{\omega(nu)}{\big(1 + H_\|^2 \big/ E^2 (nu)^2\big)^{1/2}}, \qquad \varkappa = -\frac{g\mu_0 \omega H_\| / |\mu_0| E}{\big(1 + H_\|^2 \big/ E^2 (nu)^2\big)^{1/2}}. \tag{55}$$

Therefore,

$$\dot\Omega_0 = \omega(nu)$$
$$\dot\Omega = \omega(nu)((1 + 2g\mu_0 H_\|/\omega(nu))^2 + \mathrm{d}^2)^{1/2},$$
$$(\mathbf{t}\mathbf{t}_0) = \frac{1 + 2g\mu_0 H_\|/\omega(nu)}{((1 + 2g\mu_0 H_\|/\omega(nu))^2 + \mathrm{d}^2)^{1/2}}. \tag{56}$$

If $\varkappa < \sqrt{-H^2}$, then the condition $\dot\Omega_0 < \dot\Omega$ is satisfied. In this case, the transition probability is defined by (38) and (41), and the radiation power is defined by (42) and (44).

If $\varkappa > \sqrt{-H^2}$, then the condition $\dot\Omega_0 > \dot\Omega$ is satisfied. In this case the transition probability is defined by (39) and (41), whereas the radiation power is determined by (43) and (44). For this situation to take place, it is necessary that $d \leqslant 1$ and, consequently, the constant magnetic field strength should satisfy the condition

$$q_1 < -2g\mu_0 H_\|/\omega(nu) < q_2, \tag{57}$$

where $q_{1,2} = 1 \mp \sqrt{1 - d^2}$.

The resonant case, where $\varkappa = \sqrt{-H^2}$, i.e. the condition

$$-2g\mu_0 H_\|/\omega(nu) = q_{1,2} \tag{58}$$

is satisfied, is very interesting. In this case the radiation is possible only with the frequency of the external wave and with the double frequency.

The above formulae are illustrated in figure 1. In the Redmond field one has $\mathbf{t}_0 = g\mathbf{n}$, hence the self-polarization axis processes about the wave vector of the electromagnetic wave by the angle

$$\tan\varphi = \frac{g|d|}{1 + 2g\mu_0 H_\|/\omega(nu)}. \tag{59}$$

If the condition $\sin\varphi < |d|$ is valid, self-polarization is total, hence the particle only scatters the external wave. If $\sin\varphi > |d|$ radiative transitions between states with positive and negative projections on this axis exist, frequencies of transitions being different.

It must be emphasized that the formulae for the probability of the radiative transition and radiation power are based on the solutions of equation (5). Obviously, the above deductions will also be true if we replace tensor $H^{\mu\nu}$ by any antisymmetric tensor. In [28] the spin evolution equation was deduced for a neutrino—a particle involved in the weak interaction. This equation possesses the same structure as the BMT equation; it can be obtained by the substitution of the electromagnetic field tensor $F_{\mu\nu}$ in the following way:

$$F_{\mu\nu} \to E_{\mu\nu} = F_{\mu\nu} + G_{\mu\nu}. \tag{60}$$

The tensor $G_{\mu\nu}$ describes the coherent interaction of neutrino with moving and polarized matter. In the general case of neutrino interacting with background fermions f we have

$$G^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda} g^{(1)}_{\rho} u_{\lambda} - (g^{(2)\mu} u^{\nu} - u^{\mu} g^{(2)\nu}). \tag{61}$$

Here

$$g^{(1)\mu} = \sum_f \rho^{(1)}_f j^{\mu}_f + \rho^{(2)}_f \lambda^{\mu}_f, \qquad g^{(2)\mu} = \sum_f \xi^{(1)}_f j^{\mu}_f + \xi^{(2)}_f \lambda^{\mu}_f, \tag{62}$$

where j^{μ}_f are fermion currents and λ^{μ}_f are fermion polarizations (summation is performed over all fermions f of the background). The explicit expressions for the coefficients $\rho^{(1),(2)}_f$ and $\xi^{(1),(2)}_f$ could be found if a special model of the neutrino interaction is chosen.

So if, in equation (5), we replace $H^{\mu\nu}$ by the tensor $Z^{\mu\nu} = -\frac{1}{2}\,\mathrm{e}^{\mu\nu\rho\lambda} E_{\rho\lambda}$, solutions of such an equation may be used for determining the intensity of neutrino spin light in matter and the probability of neutrino radiative transition. Only this method was used in [29, 30] for calculating characteristics of spin light within the quasi-classical approximation. If the matter density is assumed to be constant, which implies that $Z^{\mu\nu}$ is coordinate independent, it is possible to obtain the formulae for processes in matter and external constant electromagnetic fields, for example, in magnetized plasma. The results can be found by the substitution

$$H^{\mu} \to Z^{\mu} = H^{\mu} + \mu_0 (g^{(1)\mu} - u^{\mu}(g^{(1)}u)) \tag{63}$$

in equations (45)–(48), obtained for the radiation in a magnetic field.

It should be emphasized that the results obtained in this section agree in the quasi-classical region with those obtained by the methods of quantum electrodynamics in the cases when such calculations were carried out [1–6, 31].

It is of interest to compare the orders of magnitude of the radiation power I of a neutral particle and the classical radiation power of a charged particle I_0 (see, for instance, [12]). If these particles have close values of mass, e.g., a proton and a neutron, it is easy to find that

$$\frac{I}{I_0} \sim \left\{ \left(\frac{H_0}{H_{cr}}\right)^2, \left(\frac{\hbar \dot{H}_0}{mc^2 H_0}\right)^2 \right\}, \tag{64}$$

where the notations are the same as in equation (2).

Therefore, the radiation powers of a charged particle and that of a neutral particle have the same orders of magnitude either in superstrong fields, which can exist in the vicinity of astrophysical objects of pulsar type, or in very high-frequency fields.

5. Conclusions

Two important conclusions follow from the obtained results. First, neutral particles can emit radiation without spin flip. This possibility depends on the following circumstance. Particle polarization is well defined in the rest frame. To determine polarization in the laboratory frame it is necessary to make Lorentz transformation along the kinetic momentum of the particle. However, for the particle in an external field, directions of kinetic and canonical momenta, generally speaking, are different. That is why radiative transitions without spin flip do not contradict conservation laws. Therefore commonly used description of radiative transitions of neutral particles as transitions with spin flip is perfectly true only when transitions from the state which is opposite to the total self-polarization state are considered.

As a result of radiative transition, the particle always goes to the state with the spin parallel to the self-polarization axis. So even for the process in the homogeneous magnetic field, the transition probability divides into transition probabilities with and without spin flip. When the particle moves in inhomogeneous, or especially in the non-stationary electromagnetic field, the radiative transition without spin flip occurs even from the total self-polarization state. For example, this effect is inherent in the process in the plane-wave field. In this case the particle taken in the state of total polarization emits radiation with the frequency of the external wave (i.e. it scatters the external wave).

In the second place, there are configurations of external fields for which total polarization states of the particles do not exist. The example of such a configuration is the Redmond field.

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